Introduction

In this chapter, we will review basic mathematical concepts. It is important to review these concepts, even though perhaps some of you will find this rather simplistic.

Formulas as recipes

Schacht's analogy that mathematical formulas are like recipes is rather apt. If you follow a recipe in the correct order, everything will be fine; similarly, if you follow a formula in order, the math will work out right by itself.

- Ingredients become variables (age, income, etc.)
- Amounts become values (age 30, income $40,000, etc.)
- Actions become operations (addition, subtraction, multiplication, division, exponentiation, etc.)

- Finally, you have to do things in order.

Order in algebra

In algebraic functions, like recipes, we have to do things in order. The formula tells us what order to do things in. An easy way to remember the basic order is "PEMA":

- First, do operations in **p**arentheses.
- Second, do any **e**xponentiation. (x^2, y^3, etc.)
- Third, do any **m**ultiplication or division. ($x \times y$, xy, x/y, etc.)
- Finally, **a**ddition or subtraction. ($x + y$, $a - b$, etc.)

Going to the dogs. . .

Now that we know "PEMA," we can play with some data (from the chapter).

- Each dog has a "subject" or "respondent" number, to make it easier to keep track of them.
- For each dog, we collected X: each dog's desire for a bush.
- For each dog, we also collected Y: how often the bush was used by that dog.

Identifying our variables

- Our theory is that the more a dog desires a bush, the more often he/she will use a bush.
- The independent variable is X: desire for a bush.
- The dependent variable is Y: how often the bush was used.

Summation

- Take the formula "sum of X". We denote this as $\sum X$.
- $\sum X = 1 + 2 + 3 + 4 + 4 + 5 + 5 + 7 + 8 + 9 = 48$.
- We can also find $\sum Y = 2 + 2 + 3 + 5 + 7 + 9 + 11 + 11 + 12 + 13 = 75$.

More complex summations

- We can also combine summations with other operations. Note the following two formulas: $\sum X^2$ and $(\sum X)^2$

- While they appear different, they mean two different things.

- $\sum X^2$ means "the sum of all squared X values." Because of PEMA, we do exponentiation before adding!

Summating the squared X's

- So, to figure out $\sum X^2$, first we have to square all 10 X's.
- $1 \times 1 = 1$, $2 \times 2 = 4$, $3 \times 3 = 9$, $4 \times 4 = 16$, $4 \times 4 = 16$, . . .
- Now, we can add up the squared X's:
- $\sum X^2 = 1 + 4 + 9 + 16 + 16 + 25 + 25 + 49 + 64 + 81 = 290$.

Squaring the summed X's

- The other formula, $(\sum X)^2$, means something different. It means, take the sum of the X's, then square the sum. (PEMA: parentheses first!)

- So, we do what's in the parentheses first. $\sum X = 48$ (from earlier).

- Now, we exponentiate: $(\sum X)^2 = (48)^2 = 2304$.

- Note that the results are different. As we go on, we'll learn that these are important parts of bigger formulas.

A couple of bigger examples

- Now, let's look at two other formulas: $(\sum X)(\sum Y)$ and $(\sum X)^2(\sum Y)^2$.

- The first formula is pretty simple. All we have to do is multiply the two parts in parentheses together.

- From earlier, we know that $\sum X = 48$ and $\sum Y = 75$.

- So, $(\sum X)(\sum Y) = (48)(75) = 3600$.

Getting more complex

- The second formula—$(\sum X)^2(\sum Y)^2$—also has us add up the X's and Y's, but before we multiply the two sums together we have to exponentiate (remember PEMA!).

- From earlier, we know that $\sum X = 48$ and $\sum Y = 75$.

- $(\sum X)^2 = (48)^2 = 2304$ and $(\sum Y)^2 = (75)^2 = 5625$.

- So, $(\sum X)^2(\sum Y)^2 = (2304)(5625) = 12960000$ (!).

All formulas can subdivided

- All of the formulas that we look at are composed of these parts.
- For example, let's look at the formula for the sample variance (σ^2).
- $\sigma^2_Y = \frac{\sum Y^2 - \frac{(\sum Y)^2}{n}}{n-1}$.

Scary formulas

- While this formula may look big and scary, we can break it down into pieces.
- Because we have two parts, one above the line and one below it, we can break this down into the **numerator** and **denominator**.
- To figure out the division, we have to figure out both parts separately before we can figure out the whole thing.

Only three values!

First, the formula has just three values we need to figure out:

- $\sum Y^2$: the sum of the squared Y's.
- $(\sum Y)^2$: the sum of the Y's, squared.
- n: the sample size. (reminder: N is the population size).

Calculating the variance

- So, let's work from left to right on the top.

- $\sum Y^2 = (2 \times 2 + 2 \times 2 + 3 \times 3 + \cdots) = (4 + 4 + 9 + \cdots) = 727.$

- $(\sum Y)^2 = (2 + 2 + 3 + \cdots)^2 = (75)^2 = 5625.$

- n is the sample size: in this case, 10.

Now we have all the pieces. . .

- Now, let's plug these into the formula.
- $\sigma_Y^2 = \frac{\sum Y^2 - \frac{(\sum Y)^2}{n}}{n-1} = \frac{727 - \frac{5625}{10}}{10-1} = \frac{727-562.5}{9} = \frac{164.5}{9} = 18.27\overline{7}$.
- There you go!

Homework

For Monday:

- Read Box 3.
- Do the following homework questions: Ch 1 - questions 1, 4, 6, 7, 8. Ch 2 - question 2.