Answer Key

Math questions

Please show all non-trivial work. Use additional sheets if necessary.

1. According to the Mississippi Department of Transportation, the speed limit on the Holly Springs bypass is 70 m.p.h., while the mean speed (μ_x) of all traffic on the bypass is 77 m.p.h., with a standard deviation (σ_x) of 8 m.p.h. Approximately what percentage of traffic on the bypass obeys the speed limit? (i.e. What percentage of traffic travels less than 70 m.p.h.)?

   Answer:

   $$Z = \frac{X - \mu_x}{\sigma_x} = \frac{70 - 77}{8} = \frac{-7}{8} = -0.875$$

   Since we are interested in the area left of this value of Z, we obtain the small part value for $Z = 0.875$ in Appendix 1. Rounding up ($Z = 0.88$), we get a small part of .1894. So, 18.94% of traffic travels less than 70 m.p.h.

2. Approximately what is the 75th percentile speed of traffic on the bypass? (i.e. What speed does 75% of the traffic drive slower than?)

   Answer: We are interested in the big part that corresponds as close as possible to .7500. In Appendix 1, the Z that corresponds to .75 is approximately $Z = .68$.

   $$X = Z\sigma_x + \mu_x = (0.68)(8) + 77 = 82.44 \text{ m.p.h.}$$

3. What is the **probability** of correctly selecting the top two (2) finishers in order at the Indianapolis 500, if 17 cars are in the race and all of the cars have an equal chance of winning? Hint: order matters.

   Answer:

   $$nPm = \frac{N!}{(N - M)!} = \frac{17!}{(17 - 2)!} = \frac{17!}{15!} = 17 \times 16 = 272.$$

   Since there are 272 possible permutations, and we are interested in just one possibility, $p = 1/272 = .00368$.

4. What is the probability of getting four heads out of seven flips of a balanced coin? Hint: order doesn't matter.

   Answer:

   $$nCm = \frac{N!}{M!(N - M)!} = \frac{7!}{4!(7 - 4)!} = \frac{7!}{4!3!} = 7 \times 5 = 35.$$

   $$k^n = 2^7 = 128$$

   $$p = \frac{35}{128} = .2734$$

5. What is the probability of getting four or more heads out of seven flips of a balanced coin?

Answer:

$$\text{Cumulative outcomes} = \frac{7!}{4!(7-4)!} + \frac{7!}{5!(7-5)!} + \frac{7!}{6!(7-6)!} + \frac{7!}{7!(7-7)!}$$
$$= 35 + 21 + 7 + 1 = 64.$$
$$p = \frac{64}{128} = .5$$

6. The mean weight (μ_x) of members of the Ole Miss football team is 260 pounds, with a standard deviation (σ_x) of 20 pounds. We selected ten players and their mean weight ($\bar{x}$) was 295 pounds. Construct a 99% confidence interval around the **sample** mean.

Answer:

$$\mu_x = \bar{x} \pm Z_{\alpha/2}\frac{\sigma_x}{\sqrt{n}} = 295 \pm (2.58)\frac{20}{\sqrt{10}} = 295 \pm \frac{51.6}{3.162} = 295 \pm 16.32$$
$$\rightarrow 278.68 < \mu_x < 311.32$$

7. We also selected eight players on the women's volleyball team. Their mean weight ($\bar{x}$) was 140 pounds with a standard deviation (s_x) of 8 pounds. Construct a 95% confidence interval for μ_x around this sample mean.

Answer: $\alpha = .05$ and df $= 8 - 1 = 7$, so $t_{\alpha/2} = 2.365$, from Appendix 2.

$$\mu_x = \bar{x} \pm t_{\alpha/2}\frac{s_x}{\sqrt{n}} = 140 \pm (2.365)\frac{8}{\sqrt{8}} = 140 \pm \frac{2.365}{2.828} = 140 \pm 6.689$$
$$\rightarrow 133.31 < \mu_x < 146.69$$

8. Test whether or not the mean weight of our sample of 10 football players (from question 6) is likely to be the same as the team's mean weight, with a 95% confidence level. (That is, test the null hypothesis that $\bar{x} = \mu_x$ with $\alpha = .05$.)

Answer: Since $\alpha = .05$, $Z_{\text{crit}} = 1.96$.

$$Z_{\text{ob}} = \frac{\bar{x} - \mu_x}{\sigma_x / \sqrt{n}} = \frac{295 - 260}{20/\sqrt{10}} = \frac{35}{20}\sqrt{10} = 1.75\sqrt{10} = 5.53.$$

Since $Z_{\text{crit}} < Z_{\text{ob}}$, we reject the null hypothesis and conclude that the sample mean and population means differ.

9. The mean weight (μ_x) of all women on the University of Mississippi campus is 132 pounds. Test whether or not the mean weight of the sample from the women's volleyball team (from question 7) is significantly different from the mean weight of all university women, with a 99% confidence level.

Answer: Since $\alpha = .01$ and df = 8 − 1 = 7, $t_{\text{crit}} = 3.499$.

$$t_{\text{ob}} = \frac{\bar{x} - \mu_x}{s_x/\sqrt{n}} = \frac{140 - 132}{8/\sqrt{8}} = \sqrt{8} = 2.828.$$

Since $t_{\text{crit}} > t_{\text{ob}}$, we fail to reject the null hypothesis and conclude that the sample and population means are not significantly different.

10. In November 2000, an enterprising undergraduate conducted a survey of attitudes toward the presidential candidates in the 2000 presidential election on the University of Mississippi campus. She found the following results:

Group	Students (1)	Faculty (2)
n	47	38
Mean evalauation ($\bar{x}$) of Al Gore (0–100 scale)	52.3	61.7
Standard deviation (s_x)	3.8	8.2

Test the null hypothesis that $\bar{x}_1 = \bar{x}_2$; that is, test whether there was a significant difference between faculty and student evaluations of Al Gore. Use a 95% confidence level.

Answer: Since $\alpha = .05$ and df = 47 + 38 − 2 = 83, $t_{\text{crit}} = 2.000$ (from Appendix 2).

$$\begin{aligned}
s_{\text{pooled}} &= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{46(3.8)^2 + 37(8.2)^2}{83} = \frac{46(14.44) + 37(67.24)}{83} \\
&= \frac{664.24 + 2487.88}{83} = 3152.12/83 = 37.98 \\
t_{\text{ob}} &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_{\text{pooled}}(1/n_1 + 1/n_2)}} = \frac{52.3 - 61.7}{\sqrt{37.98(1/47 + 1/38)}} = \frac{-9.4}{\sqrt{37.98(0.476)}} \\
&= \frac{-9.4}{\sqrt{1.807}} = -9.4/1.34 = -6.99
\end{aligned}$$

Since $t_{\text{crit}} < |t_{\text{ob}}|$, we conclude that there is a significant difference between faculty and staff evaluations of Al Gore.

11. On September 10, seven students were asked for an evaluation (on a 0–100 scale) of George Bush. On September 12, the same seven students were again asked for an evaluation on the same scale. The data was as follows:

Subject	Evaluation on	
Number	9/10	9/12
1	72	83
2	76	92
3	67	83
4	28	57
5	87	82
6	50	76
7	17	23

Test whether the students' attitudes changed significantly, using a 95% confidence level.

Answer: Since $\alpha = .05$ and df $= 7 - 1 = 6$, $t_{\text{crit}} = 2.447$ (from Appendix 2).

$$
\begin{aligned}
\bar{d} &= \frac{\sum d}{n} = \frac{-99}{7} = -14.143 \\
s_d &= \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}} = \sqrt{\frac{2211 - \frac{(-99)^2}{7}}{7-1}} \\
&= \sqrt{810.86/6} = \sqrt{135.143} = 11.625 \\
t_{\text{ob}} &= \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{-14.143}{11.625/\sqrt{7}} \\
&= \frac{-14.143}{4.394} = -3.219
\end{aligned}
$$

Since $t_{\text{crit}} < |t_{\text{ob}}|$, we reject the null hypothesis and conclude that the difference is statistically significant.

12. On Friday night, a group of psychology students monitored the behavior of various patrons of the Burgandy Room. The number of alcoholic drinks consumed by each patron was recorded, along with the number of members of the opposite sex that the patron flirted with. The data appears in the following table:

Subject	Number of	
Number	Drinks Consumed (x)	Flirtations (y)
1	3	2
2	7	6
3	1	3
4	4	4
5	3	4
6	2	1
7	3	0
8	0	2

Assume the causal model *Consumption of alcohol* $\rightarrow$ *Flirtation*. Estimate: the slope b and intercept a of the regression line, the correlation coefficient r, and the coefficient of determination r^2. Also, test the null hypothesis that $r = 0$ with a confidence level of 95%. What does this say about our model?

Answer: Since $\alpha = .05$ and df $= 8 - 2 = 6$, $t_{\text{crit}} = 2.447$ (from Appendix 2).

$$b = \frac{\sum XY - \frac{(\sum X)(\sum Y)}{n}}{\sum X^2 - \frac{(\sum X)^2}{n}} = \frac{81 - (23)(22)/8}{97 - 23^2/8} = \frac{81 - 506/8}{97 - 529/8} = \frac{17.75}{30.875} = .575$$

$$a = \bar{y} - b\bar{x} = (22/8) - .575(23/8) = 2.75 - 1.653 = 1.097$$

$$r = \frac{\sum XY - \frac{(\sum X)(\sum Y)}{n}}{\sqrt{\sum X^2 - (\sum X)^2/n}\sqrt{\sum Y^2 - (\sum Y)^2/n}} = \frac{17.75}{\sqrt{30.875}\sqrt{86 - 22^2/8}}$$

$$= \frac{17.75}{\sqrt{30.875}\sqrt{25.5}} = \frac{17.75}{(5.56)(5.05)} = \frac{17.75}{28.059} = 0.6326$$

$$r^2 = (r)^2 = 0.6326^2 = 0.4002.$$

$$t_{\text{ob}} = r\sqrt{\frac{n-2}{1-r^2}} = 0.6326\sqrt{\frac{8-2}{1-0.4002}} = 0.6326\sqrt{6/0.6} = 0.6326\sqrt{10} = 0.6326(3.163) = 2.00$$

Since $t_{\text{crit}} > t_{\text{ob}}$, we fail to reject the null hypothesis and conclude that the correlation between x and y is not statistically significant.